Problema 1: (4 ptos)

$$B = \{\overbrace{(1,0,0,0)}^{u_1}, \overbrace{(-3,1,0,0)}^{u_2}, \overbrace{(0,5,1,0)}^{u_3}, \overbrace{(0,-2,a,b)}^{u_4}\}.$$

(1,25 pts)

1) Queremos ~~ver~~ ver primero para qué valores de a y b B es libre.

$$\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 + \alpha_4 u_4 = (0) \Rightarrow \begin{cases} \alpha_1 - 3\alpha_2 = 0 \\ \alpha_2 + 5\alpha_3 - 2\alpha_4 = 0 \\ \alpha_3 + a\alpha_4 = 0 \\ b\alpha_4 = 0 \end{cases}$$
$$= (0,0,0,0)$$

$$\left(\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & 5 & -2 & 0 \\ 0 & 0 & 1 & a & 0 \\ 0 & 0 & 0 & b & 0 \end{array}\right)$$

<u>Si $b=0$</u>, $\forall a$ ~~$\Rightarrow$ S.C.I~~ No es S.C $\Rightarrow$ El sistema no es libre $\Rightarrow$ no es Base

<u>Si $b \neq 0$</u>; $F_4 = F_4/b \Rightarrow \left(\begin{array}{cccc|c} 1 & -3 & 0 & 0 & 0 \\ 0 & 1 & 5 & -2 & 0 \\ 0 & 0 & 1 & a & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \Rightarrow \begin{cases} \alpha_4 = 0 \\ \alpha_3 + a\underset{0}{\alpha_4} = 0 \Rightarrow \alpha_3 = 0 \\ \alpha_2 + 5\alpha_3 - 2\alpha_4 = 0 \Rightarrow \alpha_2 = 0 \\ \alpha_1 - 3\alpha_2 = 0 \Rightarrow \alpha_1 = 0 \end{cases}$

Entonces si <u>$b \neq 0$</u>, el sistema B es libre en $\mathbb{R}^4$ que tiene la base canónica $B_4 \Rightarrow$ <u>B es base</u>

2) (1,25 pts)

$$M_B^{B_4}(Id) = \left[M_{B_4}^{B}(Id)\right]^{-1} = \begin{pmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 3 & -15 & 21 \\ 0 & 1 & -5 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3) (1,5 pts)

Sean $B_3 = \{(1,0,0), (0,1,0), (0,0,1)\}$ la base canónica de $\mathbb{R}^3$

$B_4 = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$,, ,, $\mathbb{R}^4$

Definimos
$$\mathbb{R}^3_{B_3} \xrightarrow{f} \mathbb{R}^4_{B_4} \xrightarrow{Id_{\mathbb{R}^4}} \mathbb{R}^4_B$$

Tenemos
$$M_B^{B_3}(f) = M_B^{B_3}(Id_{\mathbb{R}^4} \circ f) = M_B^{B_4}(Id) \cdot M_{B_4}^{B_3}(f)$$

Luego $M^{B_3}_{B_4}(f) = \left(f(1,0,0)_{B_4}, f(0,1,0)_{B_4}, f(0,0,1)_{B_4} \right)$.

Tenemos

$$f(1,0,0) = f[(1,-1,0)+(0,1,0)] = \overbrace{f\underbrace{(1,-1,0)}_{\in \ker f}}^{(0)} + f(0,1,0) = f$$

$$= (1,2,3,5)$$

$$f\underbrace{(0,0,1)}_{\in \ker f} = (0) = (0,0,0,0)$$

Entonces $M^{B_3}_{B_4}(f) = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 0 \\ 5 & 5 & 0 \end{pmatrix}$

$\Rightarrow$

$$M^{B_3}_{B}(f) = \begin{pmatrix} 1 & 3 & -15 & 21 \\ 0 & 1 & -5 & 7 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{4\times 4} \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 3 & 3 & 0 \\ 5 & 5 & 0 \end{pmatrix}_{4\times 3}$$

$$\boxed{M^{B_3}_{B}(f) = \begin{pmatrix} 67 & 67 & 0 \\ 22 & 22 & 0 \\ -2 & -2 & 0 \\ 5 & 5 & 0 \end{pmatrix}}$$